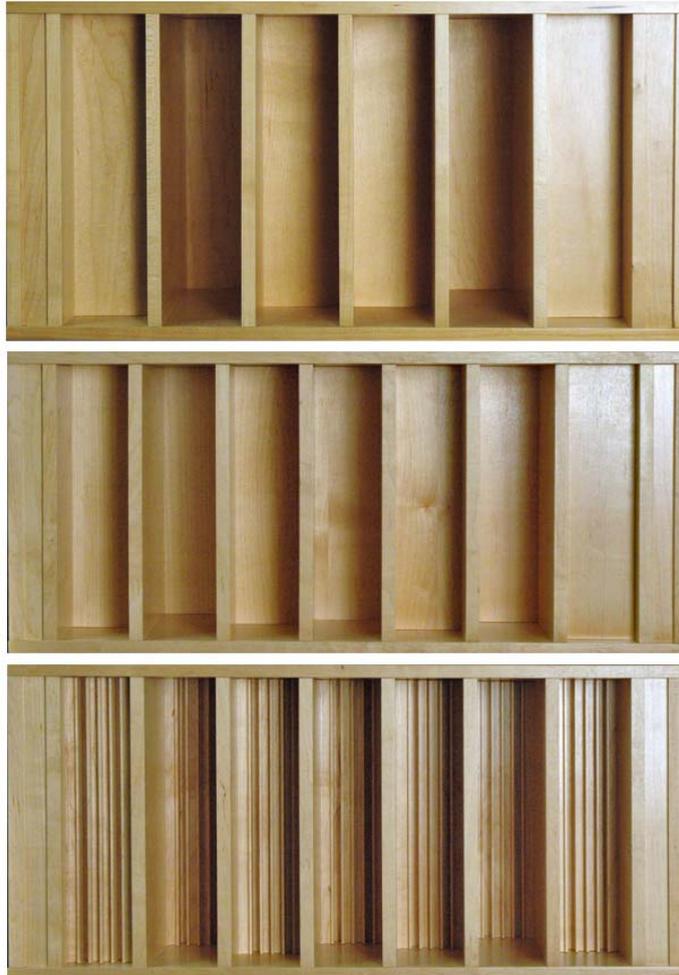# Tech Topic 01242014

Three Limitations of Traditional Number Theory Diffusors

Abstract: Traditional diffusors based on number theory like the quadratic residue diffusor (QRD®- Top) and the primitive root diffusor (PRD®) have three basic limitations, which RPG has eliminated in its development of the modulated optimized diffusor (MOD- Middle) and modulated optimized diffractal (MOF- Bottom). You should know that all diffusors in the marketplace based on number theory are 30 years behind the state of the art. RPG's current optimized diffusors are patented so you know you are getting the latest technology and best performance from RPG.

Due to the success of RPG's QRD®, older traditional absorption and noise control manufacturers, as well as startups have attempted to copy it. Specifiers should be made aware that because these manufacturers are not authoritative in sound scattering technology they are simply copying technology that is 30 years old. RPG is internationally recognized as the authority in sound diffusor design and optimization and in 2001, RPG's Founder, Dr. Peter D'Antonio and Prof. Trevor Cox published the industry's most comprehensive reference text describing the current state of knowledge on sound absorbing and sound diffusing surfaces, entitled "Sound Absorbers and Diffusers: Theory, Design and Application" Taylor & Francis Second Edition 2009. We respectfully suggest that specifiers reject outdated QRD® technology and consider the advanced Modffusor design patented by RPG to overcome all of the shortcomings of the traditional QRD. In this tech topic we describe the evolution of the number theory diffusor to illustrate the technologic advances made over the last 30 years following the introduction of the QRD.
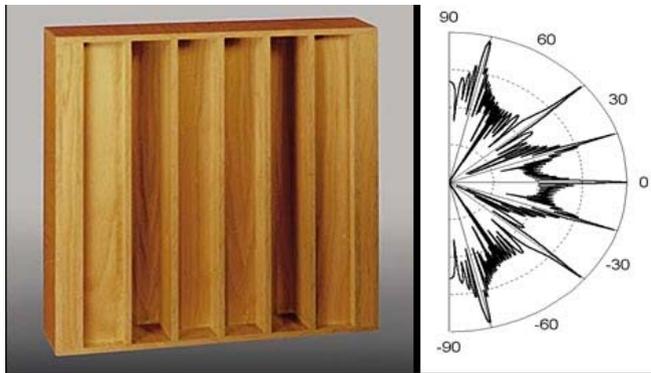

Figure 1. Left: Commercial QRD® N=7. Right: Diffraction pattern at 3,000 Hz for 50 periods where the energy is concentrated in the diffraction directions.

While the QRD, shown in Figure 1, was revolutionary when introduced by RPG Diffusor Systems three decades ago, there were three limitations that RPG methodically eliminated to improve performance, shown in Figure 2.
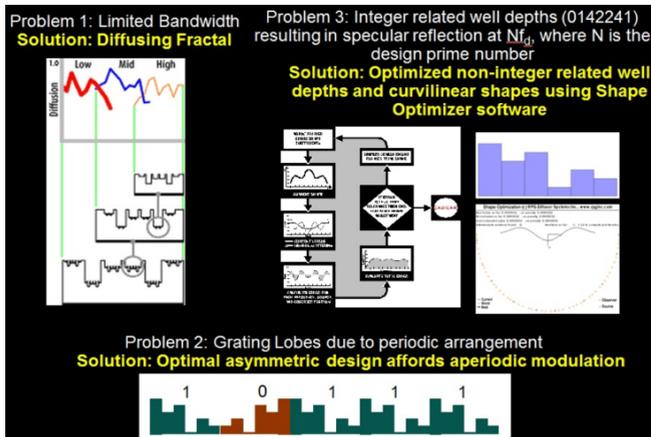

Figure 2. Three problems that were mitigated to optimize the reflection phase grating.

These include:
- Extending the bandwidth
- Minimizing the effect of grating lobes, i.e. making the response uniform, and lastly
- Eliminating the quantized well depth effect, which results in a specular reflection at the frequency where all wells scatter in phase.

Mitigating Limited Bandwidth Using Fractals

When considering how to expand the bandwidth, Dr. D'Antonio was intrigued by the idea of the self-similarity of fractals and proposed nesting, scaled versions of the QRD® forming a self-similar design, in which each generation of nesting would cover different frequency ranges [P. D'Antonio, "A new 1 or 2-dimensional fractal sound diffusor", J. Acoust. Soc. Am., Suppl. 1, 87, S10]. The nesting of scaled replicas is shown in Figure 3 for the Koch fractal and the Diffractal, along with the Hausdorff Dimension, D. The diffusing fractal or Diffractal® has been patented and
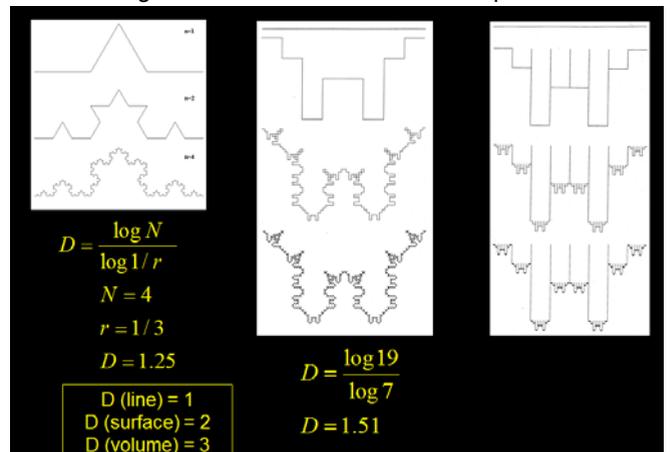

Figure 3. Fractal nested scaled replicas shown for the Koch fractal (left) and Diffractal (center without dividers and right with dividers). The corresponding Hausdorff dimension, D, is also shown.

Figure 4. Diffractal in various recording studios. Upper Left: Starstruck Studios, Nashville; Upper Right: Hit Factory, New York; Lower Left: Blackbird Studios, Nashville; Lower Right: Real World Studios, Box, UK; Center: Festival Records, Sydney.

formed the cornerstone of many celebrity recording studios, shown in Figure 4.

Minimizing the effect of grating lobes, i.e. making the response uniform using modulation

The QRD® has now been used in thousands of projects in a wide range of venues. As with any technology, research and experimentation lead to advances. The QRD® is a reflection phase grating formed from the periodic repetition of a base shape, consisting of a series of wells of depth based on the quadratic residue sequence, separated by dividers. While periodicity is the basis of the QRD®, it is also one of its limitations, because periodicity causes lobing in specific diffraction directions. The scattered polar responses in Figure 5 are dominated by grating lobes gen-
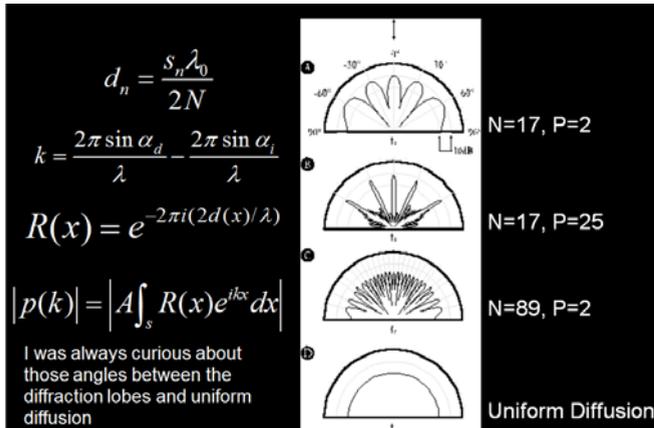

Figure 6. Illustrates how an optimized base shape, assigned a binary zero, and a flipped base shape, assigned a binary 1, can be modulated to mitigate grating lobes and provide uniform scattering.

Figure 5. Formulas for the well depths, d_n, and polar response, p(k) for periodic number theoretic diffusors based on different primes, N, and number of repeat periods, P. While the goal of the original QRD, for example, was equal energy in the diffraction directions, RPG was able to extend this to providing equal energy or uniform diffusion in all directions using modulation.

$$d_n = \frac{s_n \lambda_0}{2N}$$

$$k = \frac{2\pi \sin \alpha_d}{\lambda} - \frac{2\pi \sin \alpha_i}{\lambda}$$

$$R(x) = e^{-2\pi i (2d(x)/\lambda)}$$

$$|p(k)| = \left| A \int_s R(x) e^{ikx} dx \right|$$

erated by the fact that the diffusors are periodic. The lobe energy may be constant, but there are large minima between the lobes, except at thigh frequencies when the number of lobes becomes very large. To minimize this lobing, which compromises the uniformity of the polar response, RPG developed the Modffusor™. By contrast to the QRD®, the Modffusor™ is formed from an aperiodic array of a single, asymmetric, non-number theoretic, optimized base shape.

Instead of repeating it periodically, one would follow the prescription of an optimal binary sequence whose aperiodic Fourier transform is as flat as possible. That is, if the binary sequence is a zero, the base shape is used, if the sequence value if one, the asymmetric diffusor is flipped. In this way an aperiodic modulation is formed and grating lobes are minimized. This modulation of base shape and flipped base shape is shown in Figure 6.

In Figure 7 we show RPG's advanced Modffusor on the left in a white birch finish and the traditional QRD in a stained
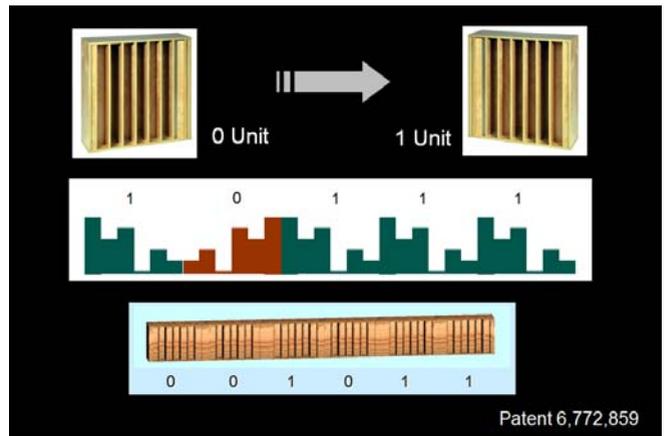

Figure 7. Left- RPG's advanced patented QRD®, which is called a Modffusor, because it's design is based on a modulated, optimized diffusor as opposed to the older number theory approach introduced in 1983. Right: Traditional number theoretic QRD®.

oak finish. The asymmetric Modffusor contains 7 full-width wells and 2 zero-depth half-width end wells. The Modffusor is no longer bound to contain a prime number of wells as in the number theoretic QRD which contains 6 full-width wells and two zero-depth, half-width end wells. These end wells are part of RPG's patented approach. The optimization and aperiodic modulation of the asymmetric Modffusor permits the unit to be 14% shallower than the QRD, with superior performance, Figure 8. For even superior performance. the fractal concept has be applied to the Modffusor forming a Modffractal, which represents the apex in diffusive performance. At the end of this description, we present the evolution of the number theoretic diffusor along with improvements in the diffusion coefficient.
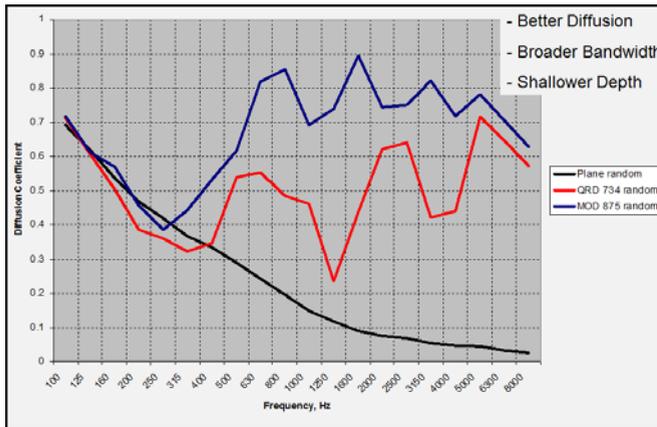
Figure 8. Diffusion response for Modffusor versus QRD. Note the significant improvement and extended low frequency bandwidth.

Eliminating the quantized well depth effect, which results in a specular reflection at the frequency where all wells scatter in phase.

For quantized wells, at N times the design frequency all wells are in phase
We developed a BEM program, Shape Optimizer, to find optimal diffusion using non-quantized wells
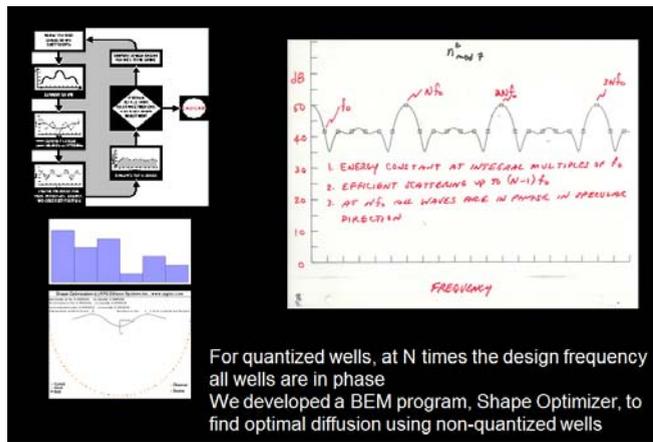
Figure 9. RPG's proprietary Shape Optimizer provides the industry's only capability to optimize the topology of rectilinear and curvilinear shapes for optimal non-number theoretic performance. Periodic reflection phase gratings become reflective at integer multiples of the design frequency, f0, times the prime, N.

Number theoretic diffusors are based on sequences based on prime numbers, which are repeated periodically. We have described the grating lobe problem caused by periodicity and now we describe the last limitation of the number theoretic diffusor, namely specular scattering at integer multiples of the design frequency, f0, times the prime. This occurs because at these frequencies all of the scattering form the various well depths, which are quantized and integerly related, is in phase, as opposed to optimally phase shifted within the diffusion bandwidth.

To mitigate this flat plate problem, RPG developed a Shape Optimizer software based on combining boundary element, multi-dimensional optimization and diffusion coefficient techniques, shown in Figure 10.

- Predict Scattered Sound
  - BEM predictions were verified against numerous experimental measurements as being extremely accurate

$$\alpha P(R) = P_z(Q,R) + \sum_q (P(q)\frac{\partial G(R,q)}{\partial n(q)} - G(R,q)\frac{\partial P(q)}{\partial n(q)})\Delta S_q$$

- Need A Metric To Evaluate The Merit of a Surface Shape
  - The Diffusion Coefficient as described in AES-4id-2001 proved to be an effective metric for uniform scattering

- Need an Intelligent Search Engine To Navigate Through the Millions of Possible Shapes
  - There are many robust search engines, we chose the downhill simplex algorithm when derivatives are not known

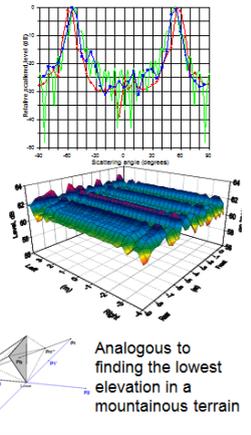Analogous to finding the lowest elevation in a mountainous terrain

Figure 10. The illustration describes how scattering is predicted using Boundary Element Techniques, how the diffusion coefficient, ISO 17497-2, is used as a metric and how the downhill simplex or genetic algorithm are used to efficiently navigate the thousands of possible shape topologies.

Summary

This review detailed the evolution of the original number theory diffusor, the QRD, introduced by RPG in 1983. It describes the three limitations of the QRD and how RPG has mitigated these limitations through fractal geometry, modulation and shape optimization. All of these advances have been patented and cannot be duplicated by competitors. RPG is also the only company providing and scattering and diffusion coefficients based on ISO 17497-1 and ISO 17497-2, respectively. In Figure 11, note the improvement in the diffusion coefficient (black) from the
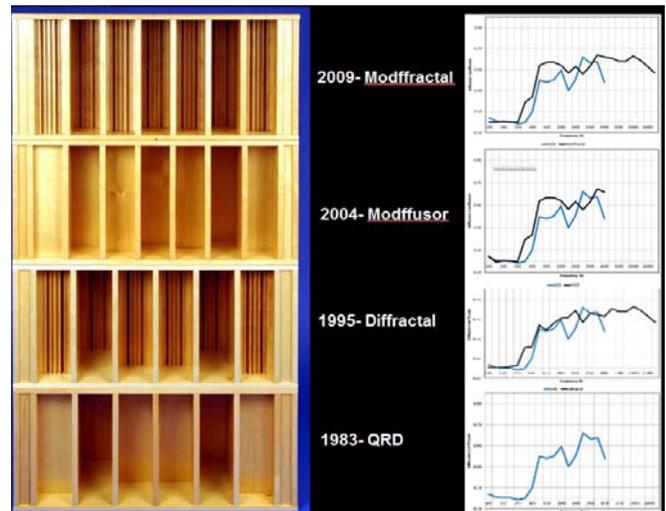
Figure 11. Comparison of the Diffusion Coefficient in the evolution from the QRD to the Modffractal.

original QRD to the Modffractal. Therefore, when choosing a diffusor, please be aware that no other company can match the performance, quality and economy of an RPG Diffusor.

4